EVIDENCE FOR FRAME-DRAGGING AROUND SPINNING BLACK HOLES IN X-RAY BINARIES

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ABSTRACT

In the context of black hole spin in X-ray binaries, we propose that certain type of quasi-period oscillations (QPOs) observed in the light curves of black hole binaries (BHBs) are produced by X-ray modulation at the precession frequency of accretion disks, due to relativistic dragging of inertial frames around spinning black holes. These QPOs tend to be relatively stable in their centroid frequencies. They have been observed in the frequency range of a few to a few hundred Hz for several black holes with dynamically determined masses. By comparing the computed disk precession frequency with that of the observed QPO, we can derive the black hole angular momentum, given its mass. When applying this model to GRO J1655-40, GRS 1915+105, Cyg X-1, and GS 1124-68, we found that the black holes in GRO J1655-40 and GRS 1915+105, the only known BHBs that occasionally produce superluminal radio jets, spin at a rate close to the maximum limit, while Cyg X-1 and GS 1124-68, typical (persistent and transient) BHBs, contain only moderately rotating ones. Extending the model to the general population of black hole candidates, the fact that only low-frequency QPOs have been detected is consistent with the presence of only slowly spinning black holes in these systems. Our results are in good agreement with those derived from spectral data, thus strongly support the classification scheme that we proposed previously for BHBs.

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1. Introduction

It has long been argued that a spinning black hole may provide an ideal laboratory for testing the theory of general relativity (GR). Recently, Zhang, Cui, & Chen (1997; Paper I hereafter) have successfully derived the angular momentum of accreting black holes (BHs) in several Galactic X-ray binaries, by carefully modeling the soft disk component in the observed X-ray spectra for high luminosity states. They found that superluminal jet sources contain BHs spinning close to the extreme theoretical limit while most others contain only slowly rotating ones. For BHBs, the relativistic dragging of inertial frames around a spinning BH should cause the accretion disk to precess, if it does not coalign with the BH equatorial plane, thus may produce direct observable effects.

Recently, Stella & Vietri (1997) proposed that for low-mass X-ray binaries that contain an accreting neutron star (LMXBNs) the observed QPOs in the range of tens of Hz are the manifestation of disk precession due to the "frame dragging" (FD) effect. They were able to account for such QPOs for several atoll and Z sources, with the help of recent discoveries of kHz QPOs in LMXBNs and progress in theoretical interpretation of such phenomena. Inspired by their work, we quickly realized that the same may also apply to certain type of QPOs that seem to be unique to BHBs.

Though not as commonly seen as in LMXBNs, QPOs have been observed in several BHBs over a frequency range from mHz to roughly ten Hz (review by van der Klis 1995, and references therein). With much improved timing resolution of the RXTE instrumentation (Bradt, Rothschild, & Swank 1993), the upper range has recently been pushed up to hundreds of Hz (Remillard et al. 1997). The QPO models that invoke magnetic effects for LMXBNs probably do not apply to BHBs, because of the less importance of the magnetic field, which is maintained by the accretion disk, in such systems. Consequently, accretion disks in BHBs can probably extend all the way (or very close) to the last stable orbit, under certain conditions. Therefore, we expect GR effects to be stronger in BHBs than in LMXBNs.

One type of QPO in BHBs is of particular interest. The QPO was first reliably established in GRS 1915+105, a superluminal jet source, with the detection of now famous 67 Hz QPO (Morgan, Remillard, & Greiner 1997). One unique characteristic of this QPO is the constancy of its centroid frequency (which moves around by only a few Hz). The QPO is clearly a transient event, and is only present in certain spectral states. The origin of such a QPO is still unknown. Shortly after this discovery, a similar QPO was detected at ∼300 Hz for GRO J1655-40, another superluminal jet source (Remillard et al. 1997). This led to the speculation that such phenomenon may be common for BHBs and may be related to some fundamental properties of these sources. In retrospect, we may have seen similar QPOs in other BHBs before as well, such as Cyg X-1 (Cui et al. 1997) and GS 1124-68 (Belloni et al. 1997), although the QPOs are not as stable (the centroid frequency can vary by up to a factor of two). A more stable QPO was also seen at 6 Hz for GX 339-4, a BH candidate (BHC), in the very high state (Miyamoto et al. 1991). It seems natural

to ask if all of these QPOs are indeed similar in origin and what are the physical processes that produce them. In this letter, we propose that they are the result of disk precession due to the FD effect. When applying this model to several known BHs with dynamically determined masses, we can derive their angular momenta. We show that the results are consistent with what we derived from spectral data in Paper I, thus providing further support to the classification scheme that we proposed for BHBs.

2. Accretion Disk and Frame Dragging Effect

As in Paper I, we assume a geometrically thin, optically thick accretion disk (Novikov & Thorne 1973; Shakura & Sunyaev 1973), which now may be tilted with respect to the equatorial plane of a spinning BH. Further more, we assume that in the high luminosity state (i.e., *not* the advection dominated state; e.g., Narayan & Yi 1995) the inner disk edge extends to the last stable orbit. Of course, with a central X-ray source, radiation pressure may have effects on the inner disk structure. We will delay the discussion of such effects until the next section.

The radius of the last (marginally) stable orbit of a test particle is a function of the BH angular momentum (Bardeen, Press, & Teukolsky 1972),

$$r_{\text{last}} = r_{\text{g}}\{3 + A_2 \pm [(3 - A_1)(3 + A_1 + 2A_2)]^{1/2}\}, \tag{1}$$

where $A_1 = 1 + (1 - a_*^2)^{1/3}[(1 + a_*)^{1/3} + (1 - a_*)^{1/3}]$, $A_2 = (3a_*^2 + A_1^2)^{1/2}$; $a_* = a/r_{\text{g}}$, with $a = J/Mc$ (J is the BH angular momentum and M the mass) and $r_{\text{g}} = GM/c^2$; the lower and upper signs are for prograde orbits (i.e., in the same sense as the BH spin, i.e., $a_* > 0$) and retrograde orbits ($a_* < 0$), respectively. In the presence of an accretion disk, a_* takes values in the range from -0.998–+0.998 (Thorne 1974), with ± 1 being the absolute theoretical limits. The event horizon of a Kerr BH is at $r_{\text{h}} = r_{\text{g}} + (r_{\text{g}}^2 - a^2)^{1/2} = r_{\text{g}}[1 + (1 - a_*^2)^{1/2}]$, so the disk may extend all the way to the horizon, $r_{\text{last}}(a_* = +1) = r_{\text{g}}$, or be expelled to $r_{\text{last}}(a_* = -1) = 9r_{\text{g}}$, as compared to the canonical Schwarzschild case, $r_{\text{last}}(a_* = 0) = 6r_{\text{g}}$.

As discussed in Paper I, X-ray emission from the disk is from the hot innermost region. The X-ray spectrum from such a disk can be described as a "diluted" blackbody, but with a varying temperature as a function of radius. The effective temperature of the disk peaks at an annulus slightly beyond r_{last} at $r_{\text{peak}} = r_{\text{last}}/\eta$, where η varies only slowly from 0.62 to 0.76 as a_* goes from -1 to +1. *That is where most of the disk X-ray emission comes from.*

Due to the FD effect, the tilted orbital plane of a test particle must precess around the same axis and in the same direction as the BH spin (Lense & Thirring 1918; Wilkins 1972). For simplicity, we specialize to only circular orbits, which are most relevant to accretion processes in X-ray binaries. First, we define a node as a point where a non-equatorial orbit intersects the equatorial plane. Then, the nodal precession frequency can be expressed as

$$\nu_{FD} = \nu_{orb}\frac{\Delta\Omega}{2\pi}, \tag{2}$$

where ν_{orb} is the orbital frequency, and $\Delta\Omega$ is an angle by which the nodes of a circular orbit are dragged per revolution. In the weak field limit, Lense and Thirring (1972) have derived $\Delta\Omega/2\pi = 2(|a|/c)(GM/r^3)^{1/2}$, where r is the radius of test particle orbit. In this case, the orbital frequency is simply that of Keplerian motion, i.e., $\nu_{orb} = (1/2\pi)(GM/r^3)^{1/2}$. Substituting these into Eq. (2), we derive the Lense-Thirring (LT) precession frequency

$$\nu_{LT} = 6.45 \times 10^4 \, |a_*| \left(\frac{M}{M_\odot}\right)^{-1} \left(\frac{r}{r_g}\right)^{-3} Hz. \tag{3}$$

For BHBs that contain extremely spinning BHs ($a_* \simeq +1$), the accretion disk can extend very close to the event horizon (at $r \simeq r_g$), where weak field approximation clearly breaks down. The exact problem has been solved analytically, by Wilkins (1972), to derive the allowed ranges for constants of motion, E, Φ, and Q, which are respectively the energy, the component of angular momentum along the BH spin axis, and a non-negative quantity related to the θ velocity (with $Q = 0$ specifying equatorial orbits). The requirement for a stable circular orbit provides two equations that allow the expression of E and Φ in terms of Q (and of course r and a_*) (cf. Wilkins 1972). Therefore, unlike the weak-field limit, for a given a_* it is no longer true in general that $\Delta\Omega$ depends only on the orbital radius.

The orbital frequency of a test particle around a spinning BH also deviates from that of Keplerian motion at small radii. Bardeen et al. (1972) have shown that the frequency is given by

$$\nu_{orb} = 3.22 \times 10^4 \left(\frac{M}{M_\odot}\right)^{-1} \left[\left(\frac{r}{r_g}\right)^{3/2} + a_*\right]^{-1} Hz. \tag{4}$$

With corrections to both terms in Eq. (2) in the case of strong field, we can compute ν_{FD} as a function of r, given a_*, Q, and BH mass. As an example, we plot the results in Fig. 1, for cases where $a_* = +0.5, +0.95, +1$, and $Q = 0.01$ (i.e., slightly off the equatorial plane). For comparison, we have also plotted the weak field approximation. As expected, strong-field effects are only important for extremely spinning BHs. Since the bulk of disk emission is produced at r_{peak}, we have derived the precession frequency at this radius, as shown in Fig. 2. It is interesting to note that the frequency is always below $\sim$8 Hz (derived for a $3M_\odot$ BH) for systems with retrograde disks, and at low frequencies there are two solutions to the BH angular momentum for a given mass.

3. Disk Precession and QPOs

We now propose that the "stable" QPOs observed in BHBs are simply X-ray modulation at the disk precession frequency. First, we apply this hypothesis to micro-quasars, GRO J1655-40 and GRS 1915+105, since we know from Paper I that both may contain nearly maximal rotating BHs. The BH mass of GRO J1655-40 was determined to a high accuracy, $7.02 \pm 0.22 M_\odot$ (Orosz

& Bailyn 1997). If the observed QPO frequency (300 Hz) is indeed the disk precession frequency at r_{peak}, we derive $a_* = +0.95$, in excellent agreement with the most probably value from Paper I. The same can be applied to GRS 1915+105, although the BH mass is not reliably determined in this case. The 67 Hz QPO implies $a_* \simeq +0.65$ for a $3M_\odot$ BH or $+0.95$ for a $30M_\odot$ one. Compared with the spectral results from Paper I, a self-consistent solution for the BH mass should be around 30 $M_\odot$. While the same value of a_* for both sources may purely be a numerical coincidence, it strongly hints at the presence of an rapidly spinning BH in GRS 1915+105 and extraordinary similarities between the two.

Then, we examine more typical BHBs. A strong QPO was observed for GS 1124-68 during the very high state (VHS) and a transition from the high to low state (Belloni et al. 1997). Within each observation, the QPO frequency remained quite stable, but it varied in the range 5–8 Hz between observations. The QPO observed during the transition (at 6.7 Hz) appears to be of the same origin. In VHS, the mass accretion rate is thought to be near the Eddington limit (van der Klis 1995), thus radiation pressure may play an important role in shaping up the inner disk structure. Depending on the exact details of the physical processes involved, VHS might not be a long-term stable state as far as the accretion disk is concerned. The inner edge of the optically thick disk may sometimes be pushed outward (Misra & Melia 1997), consequently the disk precession frequency becomes smaller. We would then expect an anti-correlation between the QPO frequency and X-ray luminosity; there seems to be a hint of such relationship for GS 1124-68 (Belloni et al. 1997). From Eq. (3) it is clear that disk precession frequency is a sensitive function of position (r) — it only takes a variation of $\sim 17\%$ in radius to account for the frequency variation observed. The BH mass of the source was measured to be $\sim 6.3M_\odot$ (Orosz et al. 1996), so a QPO frequency of 8 Hz would imply $a_* = +0.35$, which is moderate compared to that of superluminal jet sources. A similar QPO was also observed for GX 339-4 also in VHS (Miyamoto et al. 1991). The measured QPO frequency clusters around 6 Hz. It should be noted, however, that evidence for the source being a BHB is at best circumstantial — no dynamical BH mass measurement is available. Nevertheless, the source has long been considered a BHC, because of the similarities to Cyg X-1 in its X-ray properties.

Recently, a QPO has been detected in Cyg X-1 *only* during the transitions between its hard and soft states (Cui et al. 1997), perhaps similar to that in GS 1124-68 observed during the transition. The QPO frequency varied in the range $4 - 9$ Hz, showing strong correlation with X-ray spectral shape. This is consistent with the fact that the evolution of the accretion disk was not completed yet during these episodes; consequently the innermost disk may still be in the process of settling down to its final stable configuration. Given the BH mass $10M_\odot$ (Herrero et al. 1995), a QPO frequency of 9 Hz indicates $a_* = +0.48$, which is lower than what we inferred ($+0.75$) from the spectral data for the soft state. This is, nonetheless, an encouraging result considering the uncertainties (on the BH mass, the inclination angle, and so on) involved in the calculations. If the spectral state transitions are indeed caused by flip-flops of the disk, as we suggested in Paper I, we might see a QPO at ~ 2 Hz in the hard state (for $a_* = -0.48$). Of course, the detection is not

guaranteed since mechanisms for producing X-ray modulation might only be present under certain conditions. For instance, no apparent QPOs have been detected in the soft state (Cui et al. 1997).

Only low-frequency QPOs (at a few to tens of mHz) have been detected for a few other BHCs (van der Klis 1995). This seems to be consistent with most BHBs containing only slowly rotating BHs (see Paper I), although it is not clear if any of these QPOs are due to disk precession.

4. Discussion

We summarize the results for known BHBs in Table 1. Although the results are computed for a specific Q value ($Q = 0.01$), they are actually quite insensitive to Q. For example, we have calculated the angular momentum for GRO J1655-40 as a function of allowed Q values ($0 - 2$) for stable circular orbits. Over the entire range, the result varies only by roughly 1%.

Our results suggest that the FD effect is quite significant in BHBs and is manifested in the presence of relatively stable QPOs in these systems. We emphasize that the strength of the disk precession model lies in the fact that it can account for certain type of QPOs in *all* BHBs. Another model invoking trapped *g*-mode oscillations near the inner edge of the disk can also quantitatively account for the 300 Hz QPO for GRO J1655-40 and 67 Hz QPO for GRS 1915+105 (Nowak et al. 1997; Paper I), but not those in other BHBs, such as Cyg X-1 and GS 1124-68 (cf. Perez et al. 1997). Therefore, we consider the disk precession model being more natural for BHBs. The model provides a direct way of measuring BH angular momentum from the observed QPO, once the mass is determined, independent of measurements based on spectral information. It is the consistency between the two types of measurements (see Table 1) that gives us confidence in the model.

Inferring BH angular momentum directly from QPOs avoids the need for information on binary orbital inclination, thus does not suffer from the large uncertainty associated with inclination angles, which is built into spectroscopic measurements (Paper I). Compared with Paper I, a slightly lower value for a_*, from the 67 Hz QPO for GRS 1915+105, might suggest an actual inclination angle slightly larger than 70° for the system, while a higher a_* for GS 1124-68 might be due partly to a smaller inclination (than that adopted in Paper I).

The results show that the BHs in superluminal jet sources spin near the maximum limit, while those in "normal" BHBs only moderately or slowly rotate. This supports the idea that jet formation may be closely related to the BH spin. Some models suggest that jets are formed by the ejection of matter from the inner disk region and are collimated by the magnetic field maintained by the disk (e.g. Blandford & Payne 1982; Shu et al. 1995; Ustyugova et al. 1995; Kudoh & Shibata 1997). If so, the disk precession model would naturally predict the precession of jets at the same frequency in these systems. Future observations will shed light on this issue.

Why do most BHBs only contain slowly or moderately spinning BHs? There are two important spin-down mechanisms: accretion from retrograde disks onto BHs (Moderski & Sikora

1996) and the Blandford-Znajek extraction of rotational energy of spinning BHs (Blandford & Znajek 1977; Moderski, Sikora, & Lasota 1997). It has been shown that it is much easier to spin down a BH by accreting from a retrograde disk than to spin it up from a prograde disk (Thorne 1974; Moderski & Sikora 1996). For instance, a BH must accrete about 20% of its initial mass from a retrograde disk to decelerate from maximum to zero spin, while it takes roughly 180% of its current mass from a prograde disk to accelerate it back up to maximum spin. This process alone can cause a uniform initial distribution of BH angular momentum to evolve into an extremely non-uniform one with most systems being relatively slowly rotating; the Blandford-Znajek process would only speed up such evolution, despite being relatively inefficient (Moderski, Sikora, & Lasota 1997). In reality, mass accretion is probably inadequate in spinning up those born slow rotators to the extreme limit, due to such factors as low average accretion rate, disk flip-flop, or the lack of available mass from companion stars. This implies that for systems like GRO J1655-40 and GRS 1915+105 BHs are likely formed with high angular momentum.

One key element to the disk precession model is the requirement of misalignment of the disk with the equatorial plane of a spinning BH. Such misalignment could be the result of the Pringle instability (Pringle 1996; also see Stella & Vietri 1997 for detailed discussion). However, the Bardeen-Petterson effect (Bardeen & Petterson 1975) should lead to the equatorial configuration for the innermost disk, thus the precession might cease for the X-ray emitting region (although the time scale for it to happen seems to be quite long; see Scheuer & Feiler 1996). This might be the reason why the QPOs of interest only occur during transitional or unstable periods, when the inner disk region experiences significant changes, and why they only last for a limited time. Moreover, the disk precession only provides a natural frequency for QPOs, and physical mechanisms are still needed to produce X-ray modulations. At present we have no definitive knowledge about the mechanisms, but the modulation could be caused by any kind of asymmetry or inhomogeneity in a precessing disk, as well as by the varying projected area of the disk ring and/or perhaps an occulted (by the disk) central hard X-ray emitting region (cf. Stella & Vietri 1997).

Finally, it is interesting to compare BHBs with atoll sources that first drew Stella and Vietri's attention. For atoll sources, the magnetic field is thought to be very weak (e.g., van der Klis 1995), it may hardly affect the dynamics of accretion flows. Consequently, these system should resemble BHBs in their X-ray properties including QPOs. Indeed, many similarities between the two have been observed, such as hard power-law tails in the X-ray spectra (e.g., Zhang et al. 1996), which used to be considered one of the defining signatures for BHCs. It is therefore of no surprise that the disk precession model works well in both types of systems. However, no QPOs at the Keplerian frequency of the inner disk edge (appearing as kHz QPOs in LMXBNs) are present in BHBs. This might be related to the lack of magnetic field in BHBs, if the interaction between magnetosphere and accretion disk is responsible for introducing X-ray modulation. The situation becomes more complicated for LMXBNs with relatively strong field (such as Z sources), since in those cases precise knowledge about the field strength is required to determine the location of the inner disk boundary. Recent RXTE discoveries of kHz QPOs in Z sources seem to provide such

information (Stellar & Vietri 1997).

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Table 1.   Inferred Black Hole Angular Momentum*

Source	Mass ($M_\odot$)	QPO Frequency (Hz)	a_*	References
GRO J1655-40	7	300	+0.95 (+0.93)	1, 2
GRS 1915+105	30	67	+0.95 ($\sim$ +1)	3, 4
GS 1124-68	6.3	8	+0.35 (−0.04)	5, 6
Cyg X-1	10	9	+0.48 (+0.75)	7, 8

*The numbers in parentheses are the results from Paper I.

[1]Orosz & Bailyn 1997.

[2]Remillard et al. 1997.

[3]Paper I.

[4]Morgan, Remillard, & Greiner 1997.

[5]Orosz et al. 1996.

[6]Miyamoto et al. 1994.

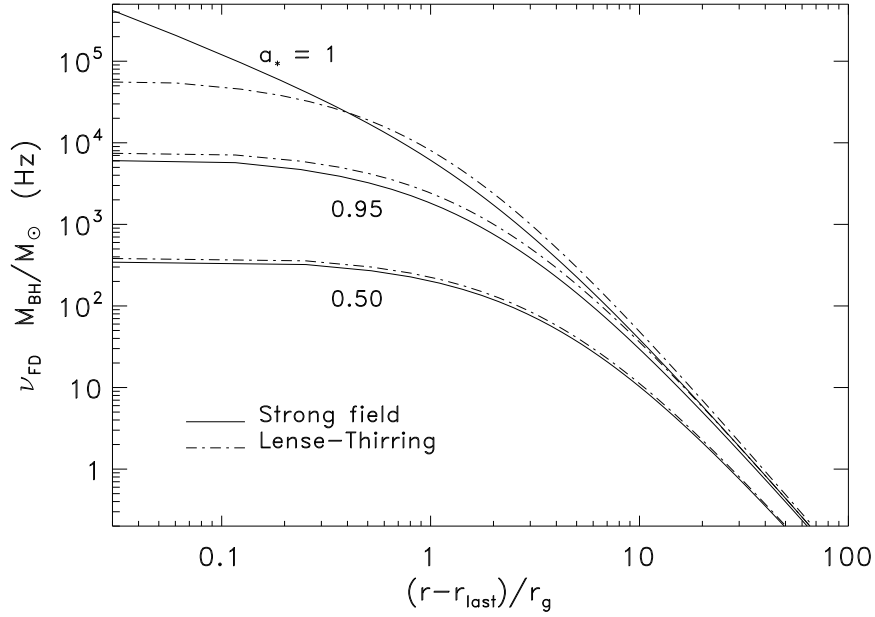[7]Herrero et al. 1995.

[8]Cui et al. 1997.

Fig. 1.— Disk precession frequency (multiplied by black hole mass), as a function of distance from the last stable orbit, for different BH angular momentum (assuming $Q = 0.01$; see text). The weak-field limits are also shown in dash-dotted line for comparison.
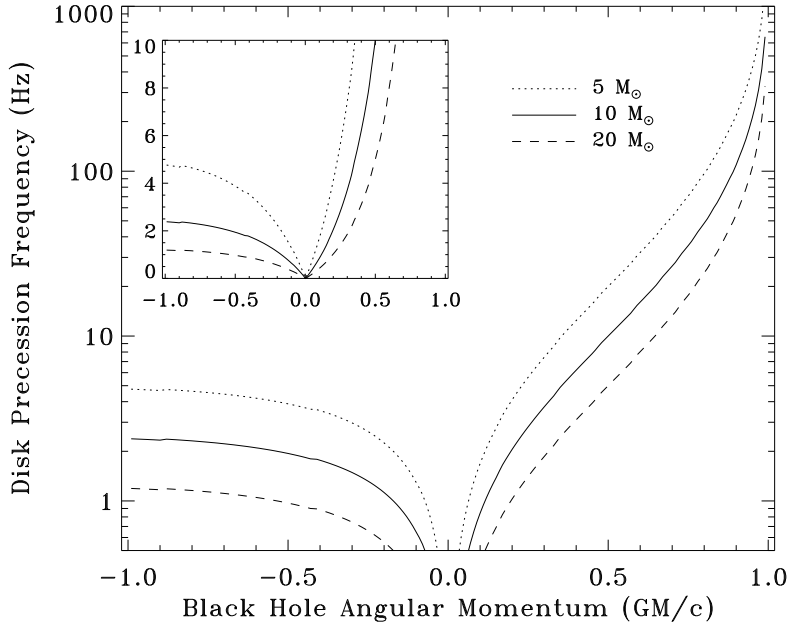
Fig. 2.— Disk precession frequency at where the disk emission peaks (r_{peak}), as a function of the dimensionless specific angular momentum (a_*) of Kerr black holes (assuming $Q = 0.01$).